

Correlations between the nuclear breathing mode energy and properties of asymmetric nuclear matter

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Based on microscopic Hartree-Fock + random phase approximation calculations with Skyrme interactions, we study the correlations between the nuclear breathing mode energy E_{ISGMR} and properties of asymmetric nuclear matter with a recently developed analysis method. Our results indicate that the E_{ISGMR} of ^{208}Pb exhibits moderate correlations with the density slope L of the symmetry energy and the isoscalar nucleon effective mass $m_{s,0}^*$ besides a strong dependence on the incompressibility K_0 of symmetric nuclear matter. Using the present empirical values of $L = 60 \pm 30$ MeV and $m_{s,0}^* = (0.8 \pm 0.1)m$, we obtain a theoretical uncertainty of about ± 16 MeV for the extraction of K_0 from the E_{ISGMR} of ^{208}Pb . Furthermore, we find the E_{ISGMR} difference between ^{100}Sn and ^{132}Sn strongly correlates with L and thus provides a potentially useful probe of the symmetry energy.

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I. INTRODUCTION

Determination of the equation of state (EOS) for isospin asymmetric nuclear matter (ANM) is among fundamental questions in both nuclear physics and astrophysics. Knowledge on the nuclear EOS is important for understanding not only the structure of finite nuclei, the nuclear reaction dynamics, and the liquid-gas phase transition in nuclear matter, but also many critical issues such as properties of neutron stars and supernova explosion mechanism in astrophysics [1–6]. In the past more than 30 years, significant progress has been made in determining the EOS of symmetric nuclear matter from subsaturation density to about 5 times normal nuclear matter density ρ_0 by studying the nuclear isoscalar giant monopole resonances (ISGMR) [7], collective flows [2] and subthreshold kaon production [8, 9] in nucleus-nucleus collisions. On the other hand, the isospin dependent part of the nuclear EOS, characterized essentially by the nuclear symmetry energy $E_{\text{sym}}(\rho)$, is still largely uncertain [5, 6]. Lack of knowledge on the symmetry energy actually hinders us to extract more accurately the EOS of symmetric nuclear matter. Therefore, to explore and narrow down the uncertainties of both the theoretical methods and the experimental data is of crucial importance for extracting more stringently information on the nuclear EOS.

During the past more than 30 years, it has been established that the nuclear ISGMR provides a good tool to probe the nuclear EOS around the nuclear normal density. In particular, it is generally believed that the incompressibility K_0 of symmetric nuclear matter can be extracted from a self-consistent microscopic theoretical model that successfully reproduces the experimental ISGMR energies as well as the ground state binding energies and charge radii of a variety of nuclei [10]. Experimentally, thanks to new and improved experimen-

tal facilities and techniques, the ISGMR centroid energy E_{ISGMR} , i.e., the so-called nuclear breathing mode energy, of ^{208}Pb (a heavy, doubly-magic nucleus with a well-developed monopole peak) has been measured with a very high precision (less than 2%). Indeed, a value of $E_{\text{ISGMR}} = 14.17 \pm 0.28$ MeV was extracted from the giant monopole resonance in ^{208}Pb based on an improved α -scattering experiment [7] (another value of $E_{\text{ISGMR}} = 13.96 \pm 0.20$ MeV was extracted in Ref. [11]). The E_{ISGMR} of ^{208}Pb has been extensively used to constrain the K_0 parameter in the literature [7, 11–21]. It is thus important to estimate and eventually narrow down the theoretical uncertainty of extracting K_0 from the nuclear ISGMR. Theoretically, in fact, it has been found that the uncertainty of the density dependence of the symmetry energy has significantly influenced the precise extraction of the K_0 parameter from ISGMR in ^{208}Pb and it also provides an explanation for the observed model dependence of the K_0 extraction from the ISGMR in ^{208}Pb based on non-relativistic and relativistic models [14, 15, 22–24].

In the present work, we estimate the theoretical uncertainty when one extracts the K_0 parameter from the nuclear ISGMR based on microscopic Hartree-Fock (HF) + random phase approximation (RPA) calculations with Skyrme interactions. In particular, we study the correlations between the ISGMR centroid energy and properties of ANM with a recently developed analysis method [25] in which instead of varying directly the 9 parameters in the Skyrme interaction, we express them explicitly in terms of 9 macroscopic quantities that are either experimentally well constrained or empirically well known. Then, by varying individually these macroscopic quantities within their known ranges, we can examine more transparently the correlation of the ISGMR centroid energy with each individual macroscopic quantity and thus estimate the theoretical uncertainty of the ISGMR centroid energy

based on the empirical uncertainties of the macroscopic quantities. Our results indicate that the density slope L of the symmetry energy and the isoscalar nucleon effective mass $m_{s,0}^*$ can significantly change the E_{ISGMR} of ^{208}Pb and the present uncertainties of L and $m_{s,0}^*$ can lead to a theoretical uncertainty of about ± 16 MeV for the extraction of K_0 . We further find the E_{ISGMR} difference between ^{100}Sn and ^{132}Sn displays a strong correlation with L and thus provides a potential probe of the symmetry energy.

II. METHODS

A. Skyrme-Hartree-Fock approach and macroscopic properties of asymmetric nuclear matter

The EOS of isospin asymmetric nuclear matter, given by its binding energy per nucleon, can be expanded to 2nd-order in isospin asymmetry δ as

$$E(\rho, \delta) = E_0(\rho) + E_{\text{sym}}(\rho)\delta^2 + O(\delta^4), \quad (1)$$

where $\rho = \rho_n + \rho_p$ is the baryon density with ρ_n and ρ_p denoting the neutron and proton densities, respectively; $\delta = (\rho_n - \rho_p)/(\rho_p + \rho_n)$ is the isospin asymmetry; $E_0(\rho) = E(\rho, \delta = 0)$ is the binding energy per nucleon in symmetric nuclear matter, and the nuclear symmetry energy is expressed as

$$E_{\text{sym}}(\rho) = \frac{1}{2!} \frac{\partial^2 E(\rho, \delta)}{\partial \delta^2} \Big|_{\delta=0}. \quad (2)$$

Around ρ_0 , the symmetry energy can be characterized by using the value of $E_{\text{sym}}(\rho_0)$ and the density slope parameter $L = 3\rho_0 \frac{\partial E_{\text{sym}}(\rho)}{\partial \rho} \Big|_{\rho=\rho_0}$, i.e.,

$$E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) + \frac{L}{3} \left(\frac{\rho - \rho_0}{\rho_0} \right) + O\left(\left(\frac{\rho - \rho_0}{\rho_0} \right)^2 \right). \quad (3)$$

In the standard Skyrme Hartree-Fock approach, the nuclear effective interaction is taken to have a zero-range, density- and momentum-dependent form [26], i.e.,

$$\begin{aligned} V_{12}(\mathbf{R}, \mathbf{r}) = & t_0(1 + x_0 P_\sigma) \delta(\mathbf{r}) \\ & + \frac{1}{6} t_3(1 + x_3 P_\sigma) \rho^\sigma(\mathbf{R}) \delta(\mathbf{r}) \\ & + \frac{1}{2} t_1(1 + x_1 P_\sigma) (\mathbf{K}'^2 \delta(\mathbf{r}) + \delta(\mathbf{r}) \mathbf{K}^2) \\ & + t_2(1 + x_2 P_\sigma) \mathbf{K}' \cdot \delta(\mathbf{r}) \mathbf{K} \\ & + i W_0(\sigma_1 + \sigma_2) \cdot [\mathbf{K}' \times \delta(\mathbf{r}) \mathbf{K}], \end{aligned} \quad (4)$$

with $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ and $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2$. In the above expression, the relative momentum operators $\mathbf{K} = (\nabla_1 - \nabla_2)/2i$ and $\mathbf{K}' = -(\nabla_1 - \nabla_2)/2i$ act on the wave function on the right and left, respectively. The quantities P_σ and σ_i denote, respectively, the spin exchange operator and Pauli

spin matrices. The σ , $t_0 - t_3$, $x_0 - x_3$ are the 9 Skyrme interaction parameters which can be expressed analytically in terms of 9 macroscopic quantities ρ_0 , $E_0(\rho_0)$, the incompressibility K_0 , the isoscalar effective mass $m_{s,0}^*$, the isovector effective mass $m_{v,0}^*$, $E_{\text{sym}}(\rho_0)$, L , the gradient coefficient G_S , and the symmetry-gradient coefficient G_V [25, 27], i.e.,

$$t_0 = 4\alpha/(3\rho_0) \quad (5)$$

$$x_0 = 3(y - 1)E_{\text{sym}}^{\text{loc}}(\rho_0)/\alpha - 1/2 \quad (6)$$

$$t_3 = 16\beta/[\rho_0^\gamma(\gamma + 1)] \quad (7)$$

$$x_3 = -3y(\gamma + 1)E_{\text{sym}}^{\text{loc}}(\rho_0)/(2\beta) - 1/2 \quad (8)$$

$$t_1 = 20C/[9\rho_0(k_F^0)^2] + 8G_S/3 \quad (9)$$

$$t_2 = \frac{4(25C - 18D)}{9\rho_0(k_F^0)^2} - \frac{8(G_S + 2G_V)}{3} \quad (10)$$

$$x_1 = \left[12G_V - 4G_S - \frac{6D}{\rho_0(k_F^0)^2} \right] / (3t_1) \quad (11)$$

$$x_2 = \left[20G_V + 4G_S - \frac{5(16C - 18D)}{3\rho_0(k_F^0)^2} \right] / (3t_2) \quad (12)$$

$$\sigma = \gamma - 1 \quad (13)$$

where $k_F^0 = (1.5\pi^2\rho_0)^{1/3}$, $E_{\text{sym}}^{\text{loc}}(\rho_0) = E_{\text{sym}}(\rho_0) - E_{\text{sym}}^{\text{kin}}(\rho_0) - D$, and the parameters C , D , α , β , γ , and y are defined as [28]

$$C = \frac{m - m_{s,0}^*}{m_{s,0}^*} E_{\text{kin}}^0 \quad (14)$$

$$D = \frac{5}{9} E_{\text{kin}}^0 \left(4 \frac{m}{m_{s,0}^*} - 3 \frac{m}{m_{v,0}^*} - 1 \right) \quad (15)$$

$$\begin{aligned} \alpha = & -\frac{4}{3} E_{\text{kin}}^0 - \frac{10}{3} C - \frac{2}{3} (E_{\text{kin}}^0 - 3E_0(\rho_0) - 2C) \\ & \times \frac{K_0 + 2E_{\text{kin}}^0 - 10C}{K_0 + 9E_0(\rho_0) - E_{\text{kin}}^0 - 4C} \end{aligned} \quad (16)$$

$$\begin{aligned} \beta = & \left(\frac{E_{\text{kin}}^0}{3} - E_0(\rho_0) - \frac{2}{3} C \right) \\ & \times \frac{K_0 - 9E_0(\rho_0) + 5E_{\text{kin}}^0 - 16C}{K_0 + 9E_0(\rho_0) - E_{\text{kin}}^0 - 4C} \end{aligned} \quad (17)$$

$$\gamma = \frac{K_0 + 2E_{\text{kin}}^0 - 10C}{3E_{\text{kin}}^0 - 9E_0(\rho_0) - 6C}. \quad (18)$$

$$y = \frac{L - 3E_{\text{sym}}(\rho_0) + E_{\text{sym}}^{\text{kin}}(\rho_0) - 2D}{3(\gamma - 1)E_{\text{sym}}^{\text{loc}}(\rho_0)} \quad (19)$$

with $E_{\text{kin}}^0 = \frac{3\hbar^2}{10m} \left(\frac{3\pi^2}{2} \right)^{2/3} \rho_0^{2/3}$ and $E_{\text{sym}}^{\text{kin}}(\rho_0) = \frac{\hbar^2}{6m} \left(\frac{3\pi^2}{2} \rho_0 \right)^{2/3}$. In the above, the isoscalar effective mass $m_{s,0}^*$ is the nucleon effective mass in symmetric nuclear matter at its saturation density ρ_0 while the isovector effective mass $m_{v,0}^*$ corresponds to the proton (neutron) effective mass in pure neutron (proton) matter at baryon number density ρ_0 (See, e.g., Refs. [26, 28]). Furthermore, G_S and G_V are respectively the gradient and

symmetry-gradient coefficients in the interaction part of the binding energies for finite nuclei defined as

$$E_{\text{grad}} = G_S(\nabla\rho)^2/(2\rho) - G_V[\nabla(\rho_n - \rho_p)]^2/(2\rho). \quad (20)$$

B. HF + continuum-RPA calculations

Since the energy of the giant monopole resonance is above the single particle continuum threshold, a proper calculation should, in principle, involve a complete treatment of the particle continuum. In the present work, we study the ISGMR of nuclei by using a microscopic HF + continuum-RPA calculations with Skyrme interactions [29]. The RPA response function is solved in the coordinate space with the proton-neutron formalism including simultaneously both the isoscalar and the isovector correlation. In this way, we can take properly into account the coupling to the continuum and the effect of neutron (proton) excess on the structure of the giant resonances in nuclei near the neutron (proton) drip lines [29].

The RPA strength distribution of ISGMR of nuclei

$$S(E_x) = \sum_n | \langle n | Q | 0 \rangle |^2 \delta(E_x - E_n) \quad (21)$$

can be obtained by using the isoscalar monopole operator

$$Q^{\lambda=0,\tau=0} = \frac{1}{\sqrt{4\pi}} \sum_i r_i^2. \quad (22)$$

Furthermore, one can define the k -th energy moment of the transition strength by

$$m_k = \int dE_x E_x^k S(E_x). \quad (23)$$

The average energy of ISGMR can be defined by the ratio between the moments m_1 and m_0 , i.e.,

$$E_{\text{ave}} = m_1/m_0. \quad (24)$$

In addition, the so-called scaling energy of ISGMR can be expressed as

$$E_{\text{sca}} = \sqrt{m_3/m_1}, \quad (25)$$

while the ISGMR energy obtained from the constrained HF approach can be written as

$$E_{\text{con}} = \sqrt{m_1/m_{-1}}. \quad (26)$$

The ISGMR energies defined by Eqs. (24)-(26) will become identical in the case of a sharp single peak exhausting 100% of the sum rule. In practice, it is found that both the experimental data and the theoretical calculations show a large width of a few MeV even in the most well-established ISGMR in ^{208}Pb . However, it is interesting to note that E_{ave} and E_{con} are rather close

within a 0.1 ~ 0.2 MeV difference even when the ISGMR peak has a large width although the scaling energy E_{sca} has a large uncertainty due to the high energy tail of monopole strength, which is always the case in experimental data (and see the theoretical results in the following). Furthermore, from the relation of the energy moments $m_{k+1}m_{k-1} \geq m_k^2$, one can obtain $E_{\text{sca}} \geq E_{\text{ave}} \geq E_{\text{con}}$. Therefore, the average energy E_{ave} is usually defined as the ISGMR centroid energy and compared between the experimental data and the theoretical calculations. It should be noted that the situation of light nuclei may be quite different from that of medium and heavy nuclei considered in the present work since the strength distribution of ISGMR for light nuclei is usually very fragmented [30–32]. It was suggested in a recent work [33] that the fragmentation of the strength distribution for the light nuclei might be explained by the clustering effects which are not considered in the present work.

III. RESULTS

In the present work, as a reference for the correlation analyses performed below with the standard Skyrme interactions, we use the MSL0 parameter set [25], which is obtained by using the following empirical values for the 9 macroscopic quantities: $\rho_0 = 0.16 \text{ fm}^{-3}$, $E_0(\rho_0) = -16 \text{ MeV}$, $K_0 = 230 \text{ MeV}$, $m_{s,0}^* = 0.8m$, $m_{v,0}^* = 0.7m$, $E_{\text{sym}}(\rho_0) = 30 \text{ MeV}$, $L = 60 \text{ MeV}$, $G_V = 5 \text{ MeV}\cdot\text{fm}^5$, and $G_S = 132 \text{ MeV}\cdot\text{fm}^5$. And the spin-orbit coupling constant $W_0 = 133.3 \text{ MeV}\cdot\text{fm}^5$ is used to fit the neutron $p_{1/2}$ - $p_{3/2}$ splitting in ^{16}O . It has been shown [25] that the MSL0 interaction can describe reasonably the binding energies and charge rms radii for a number of closed-shell or semi-closed-shell nuclei. We further find that the MSL0 parameter set predicts a value of 1.06 MeV for the splitting of the neutron $3p$ shell in ^{208}Pb , which reasonably describes the experimental value of 0.9 MeV. It should be pointed out that the MSL0 is only used here as a reference for the correlation analyses. Using other Skyrme interactions obtained from fitting measured binding energies and charge rms radii of finite nuclei as in usual Skyrme parametrization will not change our conclusion.

As numerical examples, in the present work, we choose the spherical closed-shell doubly-magic nuclei ^{208}Pb , ^{100}Sn , and ^{132}Sn . Thus, we do not include the pairing interaction since it has negligible effects on these spherical closed-shell doubly-magic nuclei considered in this work [34]. In addition, the two-body spin-orbit and the two-body Coulomb interactions are not taken into account in the present continuum-RPA calculations although the HF calculations include both of the interactions. As pointed out in Ref. [35], the net effect of the two interactions in RPA decreases the centroid energy of ISGMR in ^{208}Pb by about 300 keV. It should be stressed that, in the present work, we do not intend to extract accurately the value of the K_0 parameter by com-

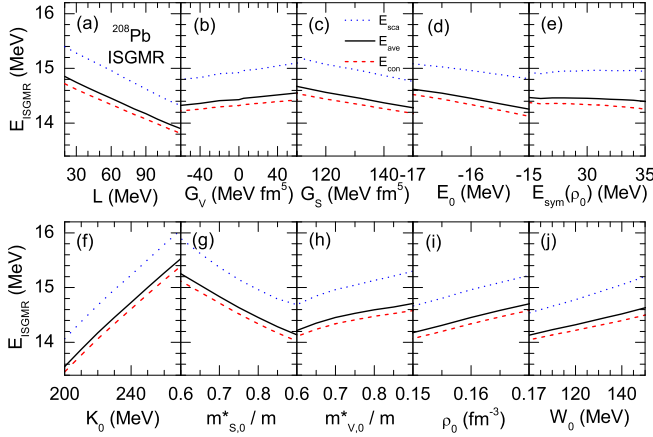


FIG. 1: (Color online) The ISGMR energies of ^{208}Pb obtained from SHF + continuum-RPA calculations with MSL0 by varying individually L (a), G_V (b), G_S (c), $E_0(\rho_0)$ (d), $E_{\text{sym}}(\rho_0)$ (e), K_0 (f), $m_{s,0}^*$ (g), $m_{v,0}^*$ (h), ρ_0 (i), and W_0 (j). The three lines from upper to lower in each panel correspond to E_{sca} , E_{ave} , and E_{con} , respectively.

paring the measured ISGMR centroid energy with that from HF + continuum-RPA calculations, and our main motivation is to explore the theoretical uncertainty for extracting K_0 . Meanwhile, we are mainly interested in the ISGMR centroid energy difference between ^{100}Sn and ^{132}Sn rather than their respective absolute value. Therefore, we do not expect that the two-body spin-orbit and the two-body Coulomb interactions in RPA will significantly change our conclusion and further work is needed to see how exactly the two interactions in continuum-RPA calculations will affect our results. Furthermore, in the following calculations, the sum rules m_k are obtained by integrating the RPA strength from excitation energy $E_x = 5$ MeV to $E_x = 35$ MeV in Eq. (23).

A. Isospin scalar giant monopole resonance in ^{208}Pb

Shown in Fig. 1 are the ISGMR energies, i.e., E_{sca} , E_{ave} , and E_{con} of ^{208}Pb obtained from SHF + continuum-RPA calculations with MSL0 by varying individually L , G_V , G_S , $E_0(\rho_0)$, $E_{\text{sym}}(\rho_0)$, K_0 , $m_{s,0}^*$, $m_{v,0}^*$, ρ_0 , and W_0 , namely, varying one quantity at a time while keeping all the others at their default values in MSL0. Firstly, one can see clearly the ordering of $E_{\text{sca}} \geq E_{\text{ave}} \geq E_{\text{con}}$ as expected. In particular, for the default parameters in MSL0, we obtain $E_{\text{sca}} = 14.962$ MeV, $E_{\text{ave}} = 14.453$ MeV, and $E_{\text{con}} = 14.338$ MeV. We note that the centroid energy of ISGMR $E_{\text{ave}} = 14.453$ MeV is essentially in agreement with the measured value of 14.17 ± 0.28 MeV for the ISGMR in ^{208}Pb [7] (a more recent experimental value of 13.96 ± 0.20 MeV was extracted in Ref. [11]). The agreement will become much better if the two-body spin-orbit and the two-

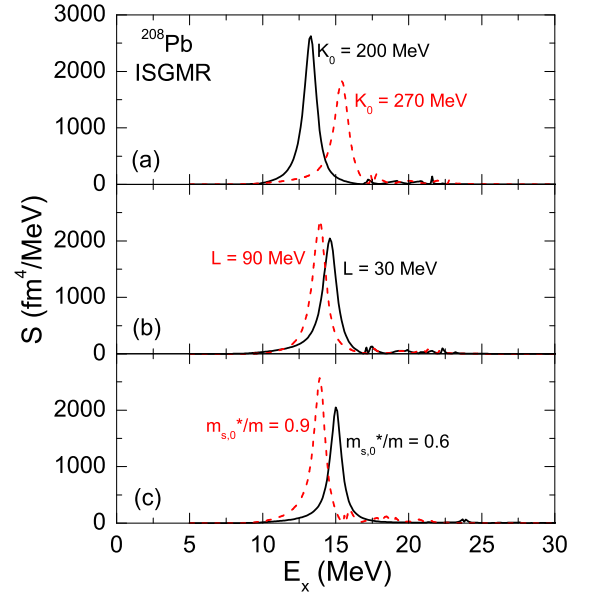


FIG. 2: (Color online) SHF + continuum-RPA response functions of ^{208}Pb with Skyrme interaction MSL0 by varying individually K_0 (a), L (b), and $m_{s,0}^*$ (c).

body Coulomb interactions are taken into account in the continuum-RPA calculations since the net effect of the two interactions in RPA reduces the centroid energy of ISGMR in ^{208}Pb by about 300 keV [35]. These features imply that the default Skyrme parameter set MSL0 can give a good description for the ISGMR in ^{208}Pb . Furthermore, one can see from Fig. 1 that, within the uncertain ranges considered here for the macroscopic quantities, the ISGMR energies display a very strong positive correlation with K_0 as expected. On the other hand, however, the ISGMR energies also exhibit moderate negative correlations with both L and $m_{s,0}^*$ while weak dependence on the other macroscopic quantities. These results indicate that the uncertainties of L and $m_{s,0}^*$ may significantly influence the extraction of K_0 by comparing the theoretical value of the ISGMR energies of ^{208}Pb from SHF + RPA calculations with the experimental measurements.

As for the correlation analysis method in Fig. 1, we would like to stress that, when the macroscopic quantities (except $E_0(\rho_0)$ and ρ_0) change individually from their base values in MSL0 within the empirical uncertain ranges considered here, the values of the binding energy or the charge rms radii of finite nuclei will vary by only about 2% at most (See, e.g., Figs. 4 and 5 in Ref. [25]). Therefore, the original agreement of MSL0 with the experimental data of binding energies or charge radii of finite nuclei will essentially still hold with the individual change of the macroscopic quantities. In this way, changing the macroscopic quantities individually in the present correlation analysis approach is equivalent to constructing a number of different Skyrme interaction sets which can give reasonable description on the ground state binding energy and charge rms radii of finite nuclei.

The key point and the most important advantage of the present analysis approach is that in the present correlation analysis, one knows exactly what is the difference among different Skyrme interaction sets constructed by using different macroscopic quantities. Furthermore, it should be mentioned that the centroid energy of ISGMR in heavy nuclei are not sensitive to the values of $E_0(\rho_0)$ and ρ_0 as shown in Fig. 1, and thus in principle we can adjust $E_0(\rho_0)$ and ρ_0 to give better description for the ground state binding energy and charge rms radii of finite nuclei without changing significantly the results of ISGMR and thus the conclusions in the present work.

In order to see the dependence of the detailed structure of ISGMR in ^{208}Pb on the values of K_0 , L and $m_{s,0}^*$, we show in Fig. 2 the SHF + continuum-RPA response functions of ^{208}Pb with MSL0 by varying individually K_0 , L , and $m_{s,0}^*$, i.e., $K_0 = 200$ and 270 MeV, $L = 30$ and 90 MeV, and $m_{s,0}^* = 0.6m$ and $0.9m$. As can be seen in Fig. 2, the RPA result displays a single collective peak in each case, consistent with the experimental data [11, 36]. Furthermore, it is seen that varying the value of K_0 from 200 MeV to 270 MeV strongly shifts the single collective peak from about 13.3 MeV to 15.4 MeV while varying the value of L ($m_{s,0}^*$) from 30 MeV ($0.6m$) to 90 MeV ($0.9m$) shifts the single collective peak from about 14.6 (15.0) MeV to 13.9 (13.9) MeV. These results are consistent with the results shown in Fig. 1. In addition, the calculated width with MSL0 by varying individually K_0 , L , and $m_{s,0}^*$ shows roughly the same value as that of experimental data [11, 36]. This agreement implies that the width of ISGMR is essentially determined by the Landau damping and the coupling to the continuum, which are properly taken into account in the present calculations.

The ISGMR energy E_{ISGMR} is conventionally related to a finite nucleus incompressibility $K_A(N, Z)$ for a nucleus with N neutrons and Z protons ($A = N + Z$) by the following definition

$$E_{\text{ISGMR}} = \sqrt{\frac{\hbar^2 K_A(N, Z)}{m \langle r^2 \rangle}}, \quad (27)$$

where m is the nucleon mass and $\langle r^2 \rangle$ is the mean square mass radius of the nucleus in the ground state. Similarly to the semi-empirical mass formula, the finite nucleus incompressibility $K_A(N, Z)$ is usually expanded as [10]

$$\begin{aligned} K_A(N, Z) = & K_0 + K_{\text{surf}} A^{-1/3} + K_{\text{curv}} A^{-2/3} \\ & + (K_\tau + K_{\text{ss}} A^{-1/3}) \left(\frac{N - Z}{A} \right)^2 \\ & + K_{\text{Coul}} \frac{Z^2}{A^{4/3}} + \dots \end{aligned} \quad (28)$$

Neglecting the curvature term K_{curv} , the surface symmetry term K_{ss} and the other higher-order terms of the finite nucleus incompressibility $K_A(N, Z)$ in Eq. (28),

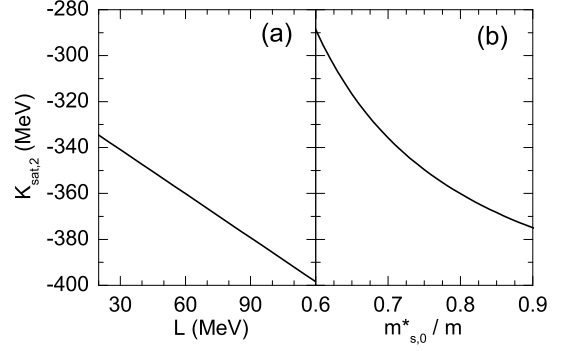


FIG. 3: The $K_{\text{sat},2}$ parameter obtained from SHF with MSL0 by varying individually L (a) and $m_{s,0}^*$ (b).

one can express $K_A(N, Z)$ as

$$\begin{aligned} K_A(N, Z) = & K_0 + K_{\text{surf}} A^{-1/3} + K_\tau \left(\frac{N - Z}{A} \right)^2 \\ & + K_{\text{Coul}} \frac{Z^2}{A^{4/3}}, \end{aligned} \quad (29)$$

where K_0 , K_{surf} , K_τ , and K_{Coul} represent the volume, surface, symmetry, and Coulomb terms, respectively. The K_τ parameter is usually thought to be equivalent to the isospin dependent part, i.e., the $K_{\text{sat},2}$ parameter, of the isobaric incompressibility coefficient of ANM (incompressibility evaluated at the saturation density of ANM) defined as

$$K_{\text{sat}}(\delta) = K_0 + K_{\text{sat},2} \delta^2 + O(\delta^4). \quad (30)$$

We would like to point out that the $K_{\text{sat},2}$ parameter is a theoretically well-defined physical property of ANM [28, 37] while the value of the K_τ parameter may depend on the details of the truncation scheme in Eq. (28) [38–42]. Here, we assume $K_{\text{sat},2}$ has similar influences on $K_A(N, Z)$ as K_τ and thus $K_{\text{sat},2}$ will affect the E_{ISGMR} through Eq. (27), and then we can analyze the L and $m_{s,0}^*$ dependences of E_{ISGMR} from the correlations of $K_{\text{sat},2}$ parameter with L and $m_{s,0}^*$.

The effects of the density dependence of the symmetry energy on the ISGMR centroid energy E_{ave} of ^{208}Pb has been extensively investigated in the literature [14, 15, 22–24]. It was firstly proposed by Piekarewicz [22] that the different symmetry energies used in the non-relativistic models and the relativistic models may be responsible for the puzzle that the former predicted an incompressibility in the range of $K_0 = 210 - 230$ MeV while the latter predicted a significantly larger value of $K_0 \approx 270$ MeV from the analysis of the ISGMR centroid energy. It is seen from Fig. 1 that a larger L value (as in usual relativistic models) leads to a smaller E_{ave} value and thus a larger K_0 value is necessary to counteract the deceasing of E_{ave} due to a larger L value. Furthermore, Fig. 1 shows that E_{ave} displays a very weak dependence on $E_{\text{sym}}(\rho_0)$, which is in contrast to the results in Ref. [15] where E_{ave}

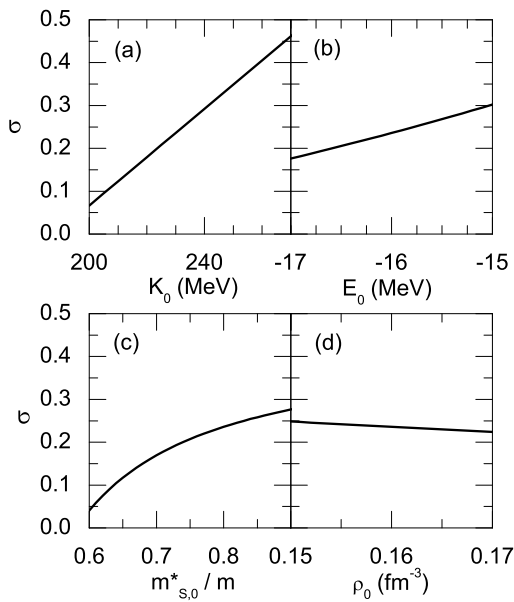


FIG. 4: The σ parameter obtained from SHF with MSL0 by varying individually K_0 (a), $E_0(\rho_0)$ (b), $m_{s,0}^*$ (c), and ρ_0 (d).

is shown to be sensitive to $E_{\text{sym}}(\rho_0)$. This is due to the fact that a constrain on the value of $E_{\text{sym}}(\rho = 0.1 \text{ fm}^{-3})$ was imposed in Ref. [15], which leads to a strong linear correlation between $E_{\text{sym}}(\rho_0)$ and L as shown recently in Ref. [43].

The symmetry energy dependence of the ISGMR centroid energy of ^{208}Pb can be understood from the fact that the ISGMR in ^{208}Pb does not constrain the compression modulus of symmetric nuclear matter but rather the one of neutron-rich matter, i.e., the isobaric incompressibility coefficient in Eq. (30). From Eq. (30) it is clear that the ISGMR in ^{208}Pb (with an isospin asymmetry of $\delta = 0.21$) should be sensitive to a linear combination of K_0 and $K_{\text{sat},2}$. The $K_{\text{sat},2}$ parameter is completely determined by the slope and curvature of the symmetry energy at saturation density as well as the third derivative of the EOS of symmetric nuclear matter (see, e.g., Ref. [28]). Fig. 3 shows the $K_{\text{sat},2}$ parameter from SHF with MSL0 by varying individually L and $m_{s,0}^*$. As can be seen in Fig. 3, the $K_{\text{sat},2}$ parameter decreases with both L and $m_{s,0}^*$, and thus $K_A(N, Z)$ for ^{208}Pb will decrease correspondingly if the $K_{\text{sat},2}$ parameter has similar effects on $K_A(N, Z)$ as the K_τ parameter and the K_{ss} term as well as the other higher-order terms in Eq. (28) are not important for $K_A(N, Z)$. These results provide an explanation on the behavior that the ISGMR energies decrease with L and $m_{s,0}^*$ observed in Fig. 1.

To understand more clearly why the ISGMR energies decrease with $m_{s,0}^*$ observed in Fig. 1, it is useful to note the fact that, with the standard Skyrme interaction, the K_0 and $m_{s,0}^*$ cannot be chosen independently if the Skyrme interaction parameter σ in Eq. (4), $E_0(\rho_0)$ and ρ_0 are fixed [44]. It should be stressed here that, instead

of assuming a fixed value of σ as in the usual parametrization and correlation analysis [15, 26], in the present work, the σ parameter is determined by four macroscopic quantities, i.e., K_0 , $E_0(\rho_0)$, $m_{s,0}^*$ and ρ_0 as shown in Eq. (13), and thus K_0 and $m_{s,0}^*$ can be chosen independently. Neglecting the isospin dependence (assuming $N \approx Z$), the nuclear breathing mode energy for medium and heavy nuclei can be approximated by [44]

$$E_{\text{ISGMR}} \approx \sqrt{\frac{\hbar^2(K_0 - 63\sigma)}{m \langle r^2 \rangle}} (K_0 \text{ in MeV}). \quad (31)$$

Eq. (31) implies that the nuclear breathing mode energy can be closely related to both K_0 and $m_{s,0}^*$ if the parameter σ is free and the values of $E_0(\rho_0)$ and ρ_0 are fixed. In Fig. 4, we show the σ parameter obtained from SHF with MSL0 by varying individually K_0 , $E_0(\rho_0)$, $m_{s,0}^*$, and ρ_0 . One can see clearly that the σ parameter indeed exhibits a strong correlation with K_0 as expected. However, it also displays a moderate dependence on $m_{s,0}^*$, a small dependence on $E_0(\rho_0)$, and a very weak correlation with ρ_0 . As can be seen in Fig. 4, the σ parameter increases with $m_{s,0}^*$, leading to smaller ISGMR energies according to Eq. (31), which is consistent with the results shown in Fig. 1. In addition, the fact that $K_{\text{sat},2}$ parameter decreases with $m_{s,0}^*$ observed in Fig. 3 will also be partially responsible for the behavior of ISGMR energies decreasing with $m_{s,0}^*$ as seen in Fig. 1 since a smaller $K_{\text{sat},2}$ value will lead to a smaller E_{ISGMR} as discussed previously.

The above results indicate that the ISGMR centroid energy of ^{208}Pb exhibits moderate correlations with both L and $m_{s,0}^*$ besides a strong dependence on K_0 . The accurate knowledge on L and $m_{s,0}^*$ is thus important for a precise determination of the K_0 parameter from the ISGMR centroid energy of ^{208}Pb . In recent years, significant progress has been made in determining L and its value is essentially consistent with $L = 60 \pm 30 \text{ MeV}$ depending on the observables and methods used in the studies [43, 45–55]. Using $L = 60 \pm 30 \text{ MeV}$, we can estimate an uncertainty of about $\pm 0.281 \text{ MeV}$ for the ISGMR centroid energy in ^{208}Pb from Fig. 1. On the other hand, for the isoscalar effective mass, the empirical value of $m_{s,0}^* = (0.8 \pm 0.1)m$ has been obtained from the analysis of both isoscalar quadrupole giant resonances data in doubly closed-shell nuclei and single-particle spectra [44, 56–59]. From Fig. 1, we can obtain an uncertainty of about $\pm 0.382 \text{ MeV}$ for the ISGMR centroid energy in ^{208}Pb using the empirical value of $m_{s,0}^* = (0.8 \pm 0.1)m$. Assuming the two uncertainties due to the present uncertainties of L and $m_{s,0}^*$ on the ISGMR centroid energy in ^{208}Pb are independent, we thus can add them quadratically to obtain an uncertainty of about $\pm 0.474 \text{ MeV}$ for the ISGMR centroid energy in ^{208}Pb . Then, using the approximate relation $(\delta K_0/K_0) = 2(\delta E_{\text{ISGMR}}/E_{\text{ISGMR}})$ from Eq. (27), we can obtain an uncertainty of $\pm 7\%$ for K_0 with $E_{\text{ISGMR}} \approx 14 \text{ MeV}$, namely, about $\pm 16 \text{ MeV}$ for $K_0 = 230 \text{ MeV}$.

Furthermore, including other uncertainties due to G_V ,

G_S , $E_0(\rho_0)$, $E_{\text{sym}}(\rho_0)$, $m_{v,0}^*$, ρ_0 and W_0 with empirical values of $G_V = 0 \pm 40$ MeV, $G_S = 130 \pm 10$ MeV, $E_0(\rho_0) = -16 \pm 1$ MeV, $E_{\text{sym}}(\rho_0) = 30 \pm 5$ MeV, $m_{v,0}^* = (0.7 \pm 0.1)m$, $\rho_0 = 0.16 \pm 0.01 \text{ fm}^{-3}$ and $W_0 = 130 \pm 20$ MeV, and assuming all the uncertainties are independent, we can obtain from Fig. 1 a total uncertainty of about ± 0.647 MeV for the ISGMR centroid energy in ^{208}Pb , which gives an uncertainty of about $\pm 9\%$ for K_0 , namely, about ± 21 MeV for $K_0 = 230$ MeV.

B. Isospin scalar giant monopole resonances in ^{100}Sn and ^{132}Sn

To see the isotopic dependence of the ISGMR centroid energy, we study here the spherical closed-shell doubly-magic nuclei ^{100}Sn and ^{132}Sn . Shown in Fig. 5 are the ISGMR centroid energy E_{ave} of ^{100}Sn and ^{132}Sn obtained from SHF + RPA calculations with MSL0 by varying individually L , G_V , G_S , $E_0(\rho_0)$, $E_{\text{sym}}(\rho_0)$, K_0 , $m_{s,0}^*$, $m_{v,0}^*$, ρ_0 , and W_0 . One can see that the results for neutron-rich nucleus ^{132}Sn are quite similar to those for ^{208}Pb as shown in Fig. 1. On the other hand, for the symmetric nucleus ^{100}Sn , it is interesting to see that the dependence of E_{ave} on the isospin relevant macroscopic quantities, namely, L , G_V , $E_{\text{sym}}(\rho_0)$, $m_{v,0}^*$ is very weak. We have also checked the case of the stable nucleus ^{90}Zr for the correlation analysis as in Fig. 5, and we find the results are very similar to the case of ^{100}Sn , namely, displaying a much weak correlation with the L parameter while a stronger correlation with the G_S parameter compared with the case of ^{208}Pb . This may be understandable from the fact that the ^{90}Zr has a smaller isospin asymmetry, i.e., $(N - Z)/A = 0.11$ compared with ^{208}Pb where we have $(N - Z)/A = 0.21$. In addition, the surface coefficient G_S may become more important for lighter nuclei as expected, leading to a stronger correlation with the G_S parameter. From these results, it seems that the ISGMR of a heavier and more symmetric nucleus, where the symmetry energy effects will be reduced significantly, may be more suitable for extracting the K_0 parameter. In addition, the different $E_{\text{ave}}-m_{s,0}^*$ correlations between ^{100}Sn and ^{132}Sn observed in Fig. 5 can be understood from the fact that $K_{\text{sat},2}$ parameter decreases with $m_{s,0}^*$ as shown in Fig. 3, leading additional decrement of E_{ave} with $m_{s,0}^*$ for the neutron-rich nucleus ^{132}Sn .

It is instructive to see the ISGMR centroid energy difference between ^{100}Sn and ^{132}Sn , which is shown in Fig. 6 with MSL0 by varying individually L , G_V , G_S , $E_0(\rho_0)$, $E_{\text{sym}}(\rho_0)$, K_0 , $m_{s,0}^*$, $m_{v,0}^*$, ρ_0 , and W_0 . It is very interesting to see from Fig. 6 that, within the uncertain ranges considered here for the macroscopic quantities, the ISGMR centroid energy difference displays a very strong correlation with L . However, on the other hand, the ISGMR centroid energy difference exhibits only moderate correlations with $m_{s,0}^*$ and $m_{v,0}^*$ while weak dependence on the other macroscopic quantities. These features imply that the ISGMR centroid energy difference between

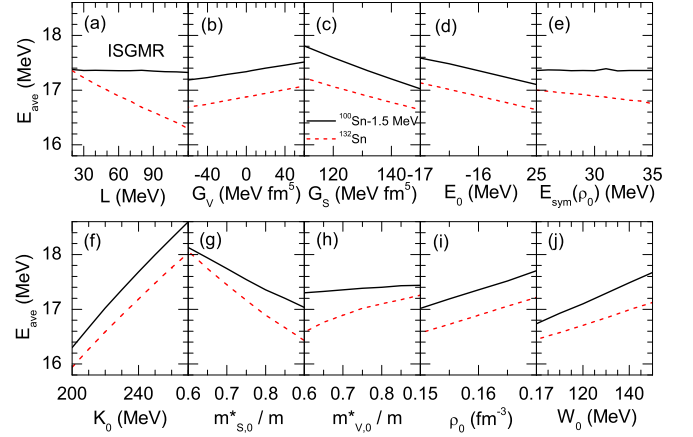


FIG. 5: (Color online) Same as Fig. 1 but for the ISGMR centroid energy E_{ave} of ^{100}Sn and ^{132}Sn . The results of ^{100}Sn shift down by 1.5 MeV for a more clear comparison with those of ^{132}Sn .

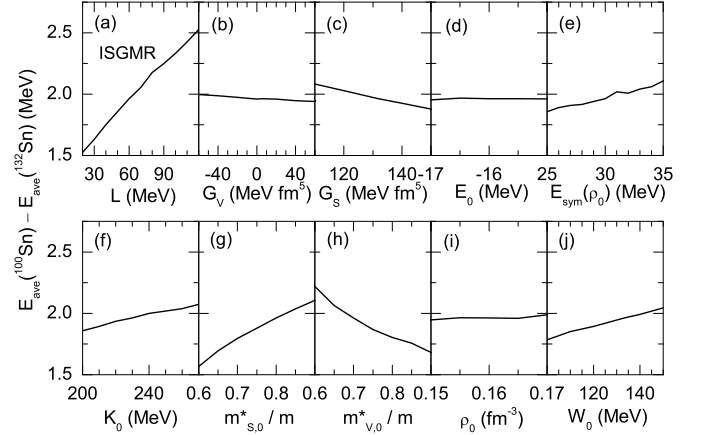


FIG. 6: Same as Fig. 1 but for the ISGMR centroid energy difference between ^{100}Sn and ^{132}Sn .

^{100}Sn and ^{132}Sn provides a potential probe of the L parameter. Furthermore, it is seen that the ISGMR centroid energy difference displays opposite correlation with $m_{s,0}^*$ and $m_{v,0}^*$, namely, increases with $m_{s,0}^*$ while decreases with $m_{v,0}^*$. Recently, a constraint of $m_{s,0}^* - m_{v,0}^* = (0.126 \pm 0.051)m$ has been extracted from global nucleon optical potentials constrained by world data on nucleon-nucleus and (p, n) charge-exchange reactions [54]. Imposing the constraint $m_{s,0}^* - m_{v,0}^* = (0.126 \pm 0.051)m$, we can expect from Fig. 6 that the correlation of the ISGMR centroid energy difference with $m_{s,0}^*$ and $m_{v,0}^*$ will become significantly weak, making the ISGMR centroid energy difference really a good probe of the L parameter. Our results indicate that a precise determination of the ISGMR centroid energy difference between ^{100}Sn and ^{132}Sn will be potentially useful to constraint accurately the symmetry energy, especially the L parameter. This provides strong motivation for measuring the ISGMR strength in unstable nuclei, which can

be investigated at the new/planning rare isotope beam facilities at CSR/HIRFL and BRIF-II/CIAE in China, RIBF/RIKEN in Japan, SPIRAL2/GANIL in France, FAIR/GSI in Germany, and FRIB/NSCL in USA.

IV. SUMMARY

The isoscalar giant monopole resonances of finite nuclei have been investigated based on microscopic Hartree-Fock + random phase approximation calculations with Skyrme interactions. In particular, we have studied the correlations between the ISGMR centroid energy, i.e., the so-called nuclear breathing mode energy, and properties of asymmetric nuclear matter within a recently developed correlation analysis method. Our results indicate that the ISGMR centroid energy of ^{208}Pb displays a very strong correlation with K_0 as expected. On the other hand, however, the ISGMR centroid energy also exhibits moderate correlation with both L and $m_{s,0}^*$ while weak dependence on the other macroscopic quantities. Using the present empirical values of $L = 60 \pm 30$ MeV and $m_{s,0}^* = (0.8 \pm 0.1)m$, we have obtained an uncertainty of about 0.474 MeV for the ISGMR centroid energy in ^{208}Pb , leading to a theoretical uncertainty of about ± 16 MeV for the extraction of K_0 from the E_{ISGMR} of ^{208}Pb . Including additionally other uncertainties due to G_V , G_S , $E_0(\rho_0)$, $E_{\text{sym}}(\rho_0)$, $m_{v,0}^*$, ρ_0 and W_0 with empirical values of $G_V = 0 \pm 40$ MeV, $G_S = 130 \pm 10$ MeV, $E_0(\rho_0) = -16 \pm 1$ MeV, $E_{\text{sym}}(\rho_0) = 30 \pm 5$ MeV, $m_{v,0}^* = (0.7 \pm 0.1)m$, $\rho_0 = 0.16 \pm 0.01 \text{ fm}^{-3}$ and $W_0 = 130 \pm 20$ MeV, we have estimated a total uncertainty of about ± 21 MeV for the extraction of K_0 by

assuming all the uncertainties are independent. These results show that the accurate knowledge on L and $m_{s,0}^*$ is important for a precise determination of the K_0 parameter by comparing the measured ISGMR centroid energy of ^{208}Pb with that from Hartree-Fock + random phase approximation calculations.

Furthermore, we have investigated how the ISGMR centroid energy difference between ^{100}Sn and ^{132}Sn correlates with properties of asymmetric nuclear matter. We have found that the ISGMR centroid energy difference between ^{100}Sn and ^{132}Sn displays a strong correlation with the L parameter while weak dependence on the other macroscopic quantities. This feature implies that the ISGMR centroid energy difference between ^{100}Sn and ^{132}Sn provides a potentially useful probe of the nuclear symmetry energy. Our results also provide strong motivation for measuring the ISGMR strength in unstable nuclei, which can be investigated at the new/planning rare isotope beam facilities around the world.

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